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Code No: 153BP

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech II Year I Semester Examinations, February - 2024
PROBABILITY AND STATISTICS AND COMPLEX VARIABLES
(Common to ME, MCT, MME, AE, MIE, PTM, TTE)

Time: 3 Hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A

(25 Marks)

1.a) Ten coins are tossed simultaneously. Find the probability that of getting at least eight heads. [2]
b) If $P(A) = P(B) = P(A \cap B)$, prove that $P(A \cap \overline{B} + \overline{A} \cap B) = 0$. [3]
c) Define exponential probability distribution. [2]
d) Find the mean and variance of the distribution $f(x) = e^{-x}, x > 0$. [3]
e) What is meant by a statistical hypothesis? [2]
f) Explain the terms i) Population ii) Parameter iii) Statistics. [3]
g) Define analytic function. [2]
h) Show that the families of curves u(x,y) = C and v(x,y) = K cut orthogonally, given that f(z) = u+iv is an analytic function. [3]
i) Find the zeros of the function $\sin\left(\frac{1}{z}\right)$. [2]
j) Expand $\log z$ by Taylor's series about $z = 1$. [3]

PART – B

(50 Marks)

2.a) If A and B toss a fair coin alternatively with the understanding that the one who obtains the head first wins. If A starts, what is the chance of winning?
b) Show that for any two events A and B, $P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$. [5+5]

OR

3.a) A can hit a target once in five shots. B can hit a target twice in 3 shots. C can hit a target once in 4 shots. What is the probability that 2 shots hit the target.
b) Suppose three companies X, Y, Z produce T.V.'s. X produces twice as many as Y while Y and Z produce the same number. It is known that 2% of X, 2% of Y and 4% of Z are defective. All the T.V.'s produced are put into one shop and then one T.V. is chosen at random. Suppose a T.V. chosen is defective, what is the probability that this T.V. is produced by companies Z and X? [5+5]

4.a) If 3 fair coins are tossed and if X is the total number of heads in the outcome. Show that X is a random variable.
b) A manufacturer of pins knows that 2% of his product is defective. If he sells pins in boxes of 100 and guarantees that not more than 4 pins will be defective. What is the probability that a box will fail to meet the guaranteed quality? [5+5]

OR

5. The marks obtained in Statistics by 1000 students are normally distributed with mean 78% and standard deviation 11%. Determine
a) how many students got marks above 90%?
b) what was the highest marks obtained by the lowest 10% of the students?
c) within what limits did the middle of 90% of the students lie? [3+3+4]

6. The owner of a machine shop must decide which of two snack vending machines to install in his shop. If each is tested 250 times, the first machine fails to work 13 times and the second machine fails to work 7 times. Test at the 0.05 level of significance whether the difference between the corresponding sample proportions is significant. [10]

OR

7. In 1950 in India the mean life expectancy was 50 years. If the life expectancies from a random sample of 11 persons are 58.2, 56.6, 54.2, 50.4, 44.2, 61.9, 57.5, 53.4, 49.7, 55.4, 57.0. Does it confirm the expected view? [10]

8.a) Show that the function $f(z) = z\bar{z}$ is differentiable but not analytic at $z = 0$.
b) Find the conjugate harmonic function of the harmonic function $u = a(1 + cos\theta)$.[5+5]

OR

9.a) If $u = e^x[(x^2 - y^2)cosy - 2xysiny]$ is the real part of an analytic function. Find the analytic function.
b) Show that xy^2 cannot be real part of any analytic function. [5+5]

10.a) Evaluate $\int_0^{1+i}(x^2 - iy)dz$ along the path $y = x^2$.
b) Evaluate $\oint_C \frac{e^z}{(z+1)^2}dz$, where C is the circle $|z - 3| = 3$. [5+5]

OR

11. Verify Cauchy's theorem for $\int z^3 dz$ over the boundary of the rectangle with vertices $-1, 1, 1 + i, -1 + i$. [10]

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